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Phemonenological model of heat transfer in an infiltrated granular bed at moderate Reynolds numbers

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Abstract

A model of heat transfer in an infiltrated granular bed, which allows for most important special features of the process, viz., anisotropy of thermal properties and nonuniform distribution of the porosity and gas (liquid) velocity over the cross section, has been formulated. The concepts of the filtration boundary layer and viscous sublayer have been introduced and identified. Temperature fields and values of the coefficient of heat exchange between the bed and the wall of the tube bounding it have been calculated. The latter are generalized in the form of the dimensionless correlation which is compared with the available experimental data. It is shown that at $Re_\infty \leqslant 2000$ the developed model describes the process of heat transfer in the granular bed well.

1. Introduction

As is known heat transfer in the infiltrated granular bed has a number of special features compared to the one-phase medium. The main of them are the anisotropy of the heat conduction coefficient, its dependence on the particle diameter and the rate of filtration, substantial difference between the structural and transport characteristics in the bed core and the region adjacent to the macrosurface bounding the bed. The joint effect of these factors imparts a specific character to the processes of heat and mass transfer, which is inherent only in a motionless granular bed. Vast literature is devoted to investigation of different aspects of this process, see, e.g., the well-known monographs [1,2]. The effect of the anisotropy of thermal properties was considered in [1,3,4]. Special features of heat transfer in the wall region, which greatly affect heat transfer, were accounted for in three different ways:

(a) By introducing near the wall an effective gas interlayer with thickness l_0 and thermal conductivity λ_{eff} dependent on the rate of filtration [5,6]. An analysis of experimental data on heat exchange of the bed with the surface within the framework of this two-layer model allowed one to obtain the following expressions for the interlayer parameters [6]:

$$l_0 = 0.1d, \tag{1}$$

$$\lambda_{\text{eff}} = A\lambda_{\text{f}} + 0.0061 c_{\text{f}}\rho_{\text{f}}u_\infty d, \tag{2}$$

where $A = 1.6$ (heat-conducting particles) and $A = 1$ (non-heat-conducting particles). The near-wall coefficient of heat transfer follows from (1) and (2) as

$$\alpha_{\text{w}} = \frac{\lambda_{\text{eff}}}{l_0} = \frac{10(A\lambda_{\text{f}} + 0.0061 c_{\text{f}}\rho_{\text{f}}u_\infty d)}{d}; \tag{3}$$

(b) By using a two-layer model in which the near-wall zone was presented in the form of an infinitely thin layer with a finite thermal resistance [1,7,8]. With such an approach, the heat conduction equation was solved at the boundary condition of the III kind

$$-\lambda_{\text{r}} \frac{\partial T}{\partial r}\bigg|_{r=R} = \alpha_{\text{w}}(T_{\text{s}} - T_{\text{w}}). \tag{4}$$

The value of the near-wall coefficient of heat transfer α_{w} was found from the comparison of model and experimental values of the heat-transfer coefficient α. It should be noted

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Nomenclature

A coefficient in (2)
$Bi = K_t R/\lambda_f$ Biot number
c_f heat capacity of gas (liquid), J/(kg K)
d particle diameter, m
K heat-transfer coefficient, W/(m^2 K)
$\tilde{K} = 1\Big/\left(\frac{d}{10(\lambda_f+0.0061 c_f \rho_f u_\infty d)} + \frac{D}{5.78(\lambda_r)_\infty} + \frac{\delta_t}{\lambda_t}\right)$ heat-transfer coefficient in [6], W/(m^2 K)
$K_t = \lambda_t/\delta_t$ W/(m^2 K)
K_0 permeability, m^2
k, k_* thickness of the filtration thermal boundary layer and thermal sublayer, m
L tube length, m
l_0 thickness of the gas film near the heat-transfer surface, m
$Nu = \alpha d/\lambda_f$, $Nu_w = \alpha_w d/\lambda_f$ Nusselt number
P pressure, kg/(m s^2)
$Pr = c_f \rho_f \nu_f/\lambda_f$ Prandtl number
$Pe = c_f \rho_f u_\infty L//\lambda_f$ Peclet number
r, $r' = r/R$ radial coordinate, m
R heat-exchanger radius, m
$Re_\infty = u_\infty d/\nu_f$ Reynolds number
$Re_{cr} = u_{cr} d/\nu_f$ critical Reynolds number
T temperature, K
T_0 temperature of the tube outer surface, K
T_w temperature of the tube inner surface, K
T_{in} gas (liquid) temperature at the inlet to the bed, K
T_s temperature at the outer surface of the near-wall zone, K
$\langle T \rangle$ cross section-mean temperature, x = const., K
u rate of gas (liquid) filtration, m/s
u_{cr} critical velocity, m/s
u_∞, $u' = u/u\infty$ rate of filtration in the bed core, m/s
x, $x' = x/L$ longitudinal (axial) coordinate, m

Greek symbols

$\alpha = 1/\left(\frac{1}{K} - \frac{1}{K_t}\right)$ heat-transfer coefficient, W/(m^2 K)
α_w near-wall coefficient of heat transfer, W/(m^2 K)
α_1, α_2 coefficients in (11), 1/s and 1/m, respectively, thickness of the filtration boundary layer, m
δ_* thickness of the filtration viscous sublayer, m
δ_t tube wall thickness, m
ε porosity
ε_∞ porosity in the bed core
$\theta = (T - T_0)/(T_{in} - T_0)$ λ thermal conductivity, W/(m K)
μ_f dynamic viscosity of gas (liquid), kg/(m s)
ν_f kinematic viscosity of gas (liquid), m^2/s
ρ_f gas (liquid) density, kg/m^3

Subscripts

a axial
cr critical
eff effective
f gas (liquid)
r radial
s particles
st stabilized
t tube wall
w near-wall
∞ bed core

that the structure of (4) logically disagrees with an assumption on an infinitely thin near-wall layer where two different temperatures T_s and T_w are possible;

(c) In the Mukhin–Smirnova model [9] it was assumed that considerable thermal resistance is concentrated near the wall in the region of a sharp temperature gradient where molecular heat transfer is realized. Assuming the thicknesses of the hydrodynamic and thermal boundary layers being equal, to calculate them we used the expression

$$\delta = k = (Re_{cr} \nu_f \sqrt{K_0})/u_\infty d, \tag{5}$$

where Re_{cr} played the role of an adjusting empirical parameter.

As is seen from the above analysis, the main feature of the mentioned models of heat transfer is their empirical character that is expressed by use of the effective parameters l_0, α_w, and Re_{cr} which are introduced *a priori* and are determined by the experimental data on the values of the heat-transfer coefficient. This indicates that being obtained in such a way they have a limited application region and cannot be justifiably used in calculations of heat transfer under new conditions.

2. Problem formulation

The paper is aimed at development of a universal model of heat transfer which adequately, from the physical point of view, describes the mechanism of the phenomenon and does not use, for this purpose, the effective quantities such as l_0, α_w, and Re_{cr}. Formulation of a model of this level presupposes:

(a) Analysis of and account for the distribution of porosity over the space of the bed;
(b) Modeling of the porosity-related distribution of gas (liquid) velocity over the bed cross section;
(c) Determination of and account for the dependences of local thermal conductivities of the granular bed on porosity, rate of filtration, thermophysical characteristics of gas (liquid) and particles;

(d) Analysis of the values of thermal conductivities in the near-wall region.

3. Heat transfer model

For definiteness, we consider heat transfer in a tube with a granular bed of the boundary condition of the first kind on the outer surface of the tube. In one-temperature approximation and with account of thermal anisotropy of the bed the heat conduction equation has the form of the equation of the 2nd order of the elliptic type

$$c_f\rho_f u(r)\frac{\partial T}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_r(r)\frac{\partial T}{\partial r}\right) + \lambda_a(r)\frac{\partial^2 T}{\partial x^2}, \tag{6}$$

the boundary conditions are

$$x = 0, \quad c_f\rho_f u_\infty T_{in} = c_f\rho_f u(r)T - \lambda_a(r)\frac{\partial T}{\partial x}, \tag{7}$$

$$x = L, \quad \frac{\partial T}{\partial x} = 0, \tag{8}$$

$$r = 0, \quad \frac{\partial T}{\partial r} = 0, \tag{9}$$

$$r = R, \quad -\lambda_r(r)\frac{\partial T}{\partial r} = \frac{\lambda_t}{\delta_t}(T - T_0). \tag{10}$$

We note that the necessity to apply the Dankwerst conditions to Eq. (6) is shown in [10]. Eq. (6) and condition (7) are written without regard for the zone of hydrodynamic stabilization of gas (liquid) flow, which, as is known, amounts to several particle diameters [11]. This allows one to consider the quantities $u(r)$, $\lambda_r(r)$, and $\lambda_a(r)$ as functions only of radius r.

Despite a rather standard form of Eq. (6) with conditions (7)–(10), calculation of temperature fields with the help of them is rather difficult due to the existing problem of correct determination of the functions $u(r)$, $\lambda_r(r)$, and $\lambda_a(r)$. The solution of it is the main task of the present study.

3.1. Distribution of gas (liquid) velocity across the tube

To calculate this distribution we used the filtration equation in the form [12]

$$-\frac{\partial P}{\partial x} = (\alpha_1 + \alpha_2 u)\rho_f u - \mu_f\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right). \tag{11}$$

The coefficients α_1 and α_2 were calculated by the formulas

$$\alpha_1 = 150\frac{(1-\varepsilon)^2}{\varepsilon^3}\frac{\nu_f}{d^2}, \quad \alpha_2 = 1.75\frac{1-\varepsilon}{\varepsilon^3}\frac{1}{d}, \tag{12}$$

which agree with the known Ergun formula [1]

$$-\frac{\partial P}{\partial x} = 150\frac{(1-\varepsilon)^2}{\varepsilon^3}\frac{\mu_f u}{d^2} + 1.75\frac{(1-\varepsilon)}{\varepsilon^3}\frac{\mu_f u^2}{d}. \tag{13}$$

The value of the pressure gradient in Eq. (1), which can be called the generalized Brinkman equation, was calculated by (13) under the condition of the bed core

$$-\frac{\partial P}{\partial x} = 150\frac{(1-\varepsilon_\infty)^2}{\varepsilon_\infty^3}\frac{\mu_f u_\infty}{d^2} + 1.75\frac{1-\varepsilon_\infty}{\varepsilon_\infty^3}\frac{\mu_f u_\infty^2}{d}. \tag{14}$$

With this in mind, the distribution of gas (liquid) velocity in the bed cross section, determined by (11), was found numerically from the solution of the boundary-value problem

$$150\frac{(1-\varepsilon_\infty)^2}{\varepsilon_\infty^3} + 1.75\frac{(1-\varepsilon_\infty)}{\varepsilon_\infty^3}Re_\infty - 150\frac{(1-\varepsilon)^2}{\varepsilon^3}\left(\frac{u}{u_\infty}\right) - 1.75\frac{1-\varepsilon}{\varepsilon^3}Re_\infty\left(\frac{u}{u_\infty}\right) + \left(\frac{d}{R}\right)^2\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial}{\partial r'}\left(\frac{u}{u_\infty}\right)\right) = 0, \tag{15}$$

$$\left.\frac{u}{u_\infty}\right|_{r'=1} = 0; \quad \left.\frac{\partial u}{\partial r'}\right|_{r'=0} = 0. \tag{16}$$

Moreover, the velocity profile based on the Ergun equation (13) was found by the formula following from (13) and (14)

$$\frac{u}{u_\infty} = 150\frac{150(1-\varepsilon)}{3.5Re_\infty} \times\left(-1+\sqrt{1+\frac{7\varepsilon^3}{150^2(1-\varepsilon)^3}\left(150\frac{(1-\varepsilon_\infty)^2}{\varepsilon_\infty^3}Re_\infty + 1.75\frac{1-\varepsilon_\infty}{\varepsilon_\infty^3}Re_\infty^2\right)}\right). \tag{17}$$

In calculations of the distributions $u(r)$ we used the dependence $\varepsilon(r)$ in the form

$$\varepsilon(r) = \varepsilon_\infty + (1-\varepsilon_\infty)\cos\left(2\pi\frac{R-r}{d}\right)\exp\left(-1.5\frac{R-r}{d}\right), \tag{18}$$

obtained as a result of generalization of experimental data of [13] by the measurements of $\varepsilon(r)$.

The results of calculation of the functions $u(r)$ by (15)–(17) are shown in Fig. 1. As is seen, the functions determined by the generalized Brinkman equation (15) fairly well agree with the functions, calculated by (17), everywhere except a narrow near-wall zone with a thickness of about $0.1d$, where the effect of gas (liquid) viscosity manifests itself. This allows one to introduce and identify, as shown in Fig. 1, the concepts of a filtration hydrodynamic boundary layer and a viscous sublayer. As is seen, a special character of the function $u(r)$ inherent only in the granular bed is realized in the filtration boundary layer. The quantities δ and δ_* calculated by the technique shown in Fig. 1, are presented in Fig. 2. According to classification of [14], three flow models were considered: laminar $5 < Re_\infty < 80$, transient $80 < Re_\infty < 120$, and turbulent $Re_\infty > 120$. The following approximation relation for δ_* is obtained for a laminar region:

$$\frac{\delta_*}{d} = 0.12Re_\infty^{-0.08}. \tag{19}$$

We have for the transient region

$$\frac{\delta_*}{d} = 0.34Re_\infty^{-0.32}. \tag{20}$$

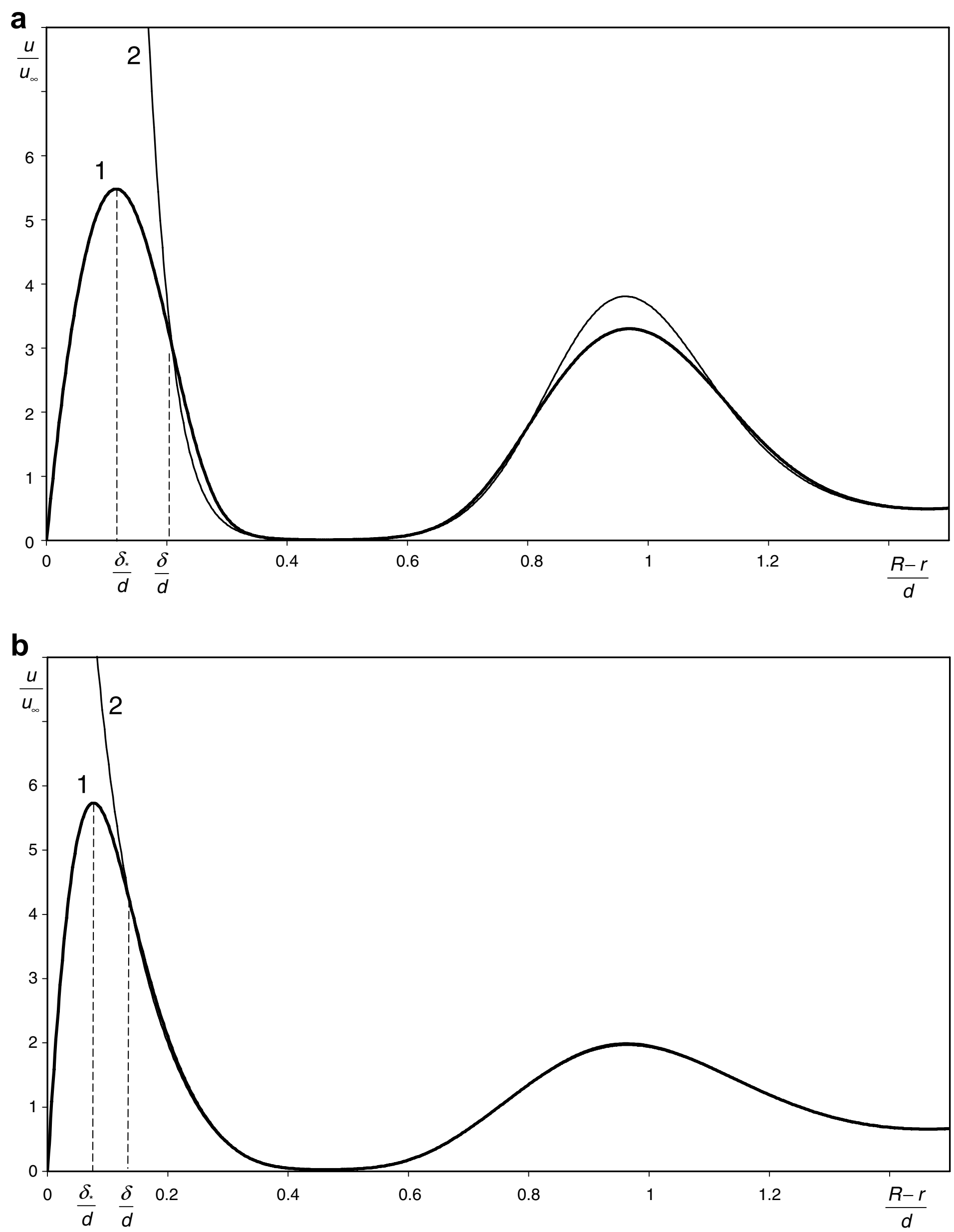

Fig. 1. Distribution of local gas (liquid) velocity in the near-wall region of the granular bed: (a) $Re_\infty = 1$, (b) 100. (1) calculation by (15) and (16); (2) calculation by (17).

and for the turbulent region

$$\frac{\delta_*}{d} = 0.33 Re_\infty^{-0.31}. \tag{21}$$

The relation

$$\frac{\delta}{\delta_*} \cong 1.78, \tag{22}$$

which holds for the three regions, was obtained for calculation of δ.

3.2. Calculation of thicknesses of the filtration thermal boundary layer and thermal sublayer

We use the known relation between δ_* and k^* [15]

$$\frac{k_*}{\delta_*} = \frac{1}{Pr^{1/3}}. \tag{23}$$

With account for (23) from (19)–(21) we obtain

a laminar region

$$\frac{k_*}{\delta_*} = 0.12 Re^{-0.08} Pr^{-0.33}, \tag{24}$$

a transient region

$$\frac{k_*}{\delta_*} = 0.34 Re^{-0.32} Pr^{-0.33}, \tag{25}$$

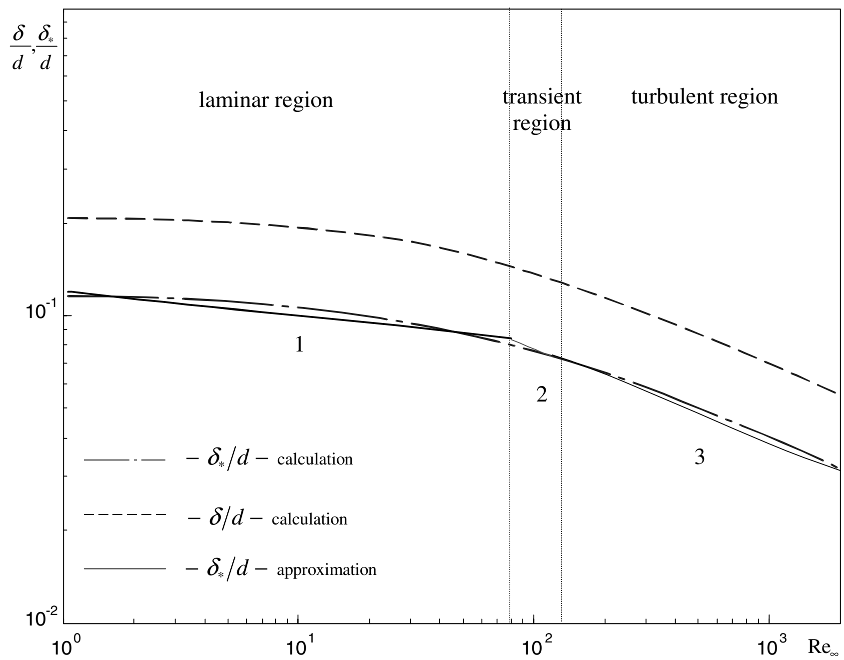

Fig. 2. Values of the filtration boundary layer and viscous sublayer. (1) relation (19); (2) (20); (3) (21).

a turbulent region

$$\frac{k_*}{\delta_*} = 0.33Re^{-0.31}Pr^{-0.33}, \tag{26}$$

The values of the thermal filtration layer were calculated by the relation $k/k_* = 1.78$ which is similar to that found earlier for δ/δ_* [formula (22)].

3.3. Determination of efficient thermal conductivities

At a distance from the heat transfer surface (in the bed core) these quantities are calculated by the formulas [1,11]

$$(\lambda_r)_\infty = \lambda_s^0 + 0.1c_f\rho_f u_\infty d, \tag{27}$$

where

$$\frac{\lambda_s^0}{\lambda_f} = 1 + \frac{(1-\varepsilon)(1-\lambda_f/\lambda_s)}{\lambda_f/\lambda_s + 0.028\varepsilon^{0.63}(\lambda_s/\lambda_f)^{0.18}}, \tag{28}$$

$$\frac{(\lambda_a)_\infty}{(\lambda_r)_\infty} = \begin{cases} 1, & Re_\infty \leqslant 10, \\ 0.66Re_\infty^{0.32}, & Re_\infty > 10. \end{cases} \tag{29}$$

We note that relation (29) was obtained as a result of processing of experimental data by $(\lambda_a)_\infty/(\lambda_r)_\infty$ given in [1].

Adaptation of relations (27)–(29) for calculation of thermal conductivities near the wall can be made with account for the existence of the thermal boundary layer and thermal sublayer. The expression for $\lambda_r(r)$ was formulated in the form

$$\lambda_r(r) = \begin{cases} \lambda_{eff}, & R - k_* < r \leqslant R, \\ \lambda_{eff} + \frac{\lambda_r^* - \lambda_f}{k - k_*}(R - k_* - r), & R - k < r < R - k_* \\ \lambda_r^* = \lambda_s^0 + 0.1c_f\rho_f u(r)d, & 0 \leqslant r < R - k. \end{cases} \tag{30}$$

To calculate $\lambda_r(r)$ we used the relation similar to (29)

$$\frac{\lambda_a(r)}{\lambda_r(r)} = \begin{cases} 1, & Re_\infty \leqslant 10, \\ 0.666Re_\infty^{0.32}, & Re > 10. \end{cases} \tag{31}$$

The value of the effective thermal conductivity in the thermal sublayer is given by relation (2) which is similar to (27).

4. Analysis of the theoretical model

We write the system of Eqs. (6)–(10) in the dimensionless form

$$Peu'(r')\frac{\partial\theta}{\partial x'} = \left(\frac{L^2}{R}\right)\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\lambda_r(r')}{\lambda_f}\frac{\partial\theta}{\partial r'}\right) + \frac{\lambda_a(r')}{\lambda_f}\frac{\partial^2\theta}{\partial(x')^2}. \tag{32}$$

The boundary conditions are

$$x' = 0, \quad \frac{T_{in}}{T_{in} - T_0} = u'(r')\left(\theta + \frac{T_0}{T_{in} - T_0}\right) - \frac{1}{Pe}\frac{\partial\theta}{\partial x'}, \tag{33}$$

$$x' = 1, \quad \frac{\partial\theta}{\partial x'} = 0, \tag{34}$$

$$r' = 0, \quad \frac{\partial\theta}{\partial r'} = 0, \tag{35}$$

$$r' = 1, \quad -\frac{\partial\theta}{\partial r'} = \frac{Bi\theta}{1 + 0.0061Re_\infty Pr}. \tag{36}$$

For comparison we considered the parabolic heat conduction equation (without regard for longitudinal thermal conductivity) with the corresponding boundary conditions

$$Pe u'(r')\frac{\partial\theta}{\partial x'} = \left(\frac{L}{R}\right)^2 \frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\lambda_r(r')}{\lambda_f}\frac{\partial\theta}{\partial r'}\right), \tag{37}$$

$$x' = 0, \quad \theta = 1, \tag{38}$$

$$r' = 0, \quad \frac{\partial\theta}{\partial r'} = 0, \tag{39}$$

$$r' = 1, \quad -\frac{\partial\theta}{\partial r'} = \frac{Bi\theta}{1 + 0.0061 Re_\infty Pr}. \tag{40}$$

The heat-transfer coefficient was calculated by the formula

$$K = -\lambda_r \frac{\partial T}{\partial r}\bigg|_{r=R} \frac{1}{\langle T\rangle - T_0}. \tag{41}$$

Fig. 3 shows the temperature fields calculated by (32)–(36) and (37)–(40) for the following parameters of the granular bed: $T_{in} = 373$ K, $T_0 = 273$ K, $L = 0.025$ m, $R = 0.005$ m, $\delta_t = 0.002$ m, $d = 0.001$ m, $\lambda_s^0 = 0.1$ W/(m K), $c_f = 1015$ J/(kg K), $A = 1$, $\lambda_t = 62$ W/(m K), $\lambda_f = 0.027$ W/(m K), $\mu_f = 1.8 \times 10^{-5}$ kg/(m s). As is seen, in all cases, rather large temperature gradients due to the presence of the thermal sublayer are observed in the near-wall region. At small Re_∞, the difference between the solutions of the elliptic and parabolic heat conduction equations are rather appreciable (Fig. 3a and b) and decrease with an increase of Re_∞ (Fig. 3c and d).

Fig. 4 shows the calculated variations of the heat-transfer coefficient along the tube length. On generalization of the obtained values of α_{st} in the case of stabilized heat transfer an important conclusion was drawn within the context of the present paper: the calculated values of α_{st} are approximated by the formula

$$Nu_{st} = \frac{1}{\frac{0.1\lambda_r}{\lambda_{eff}} + 0.345\frac{\lambda_f}{(\lambda_r)_\infty}\frac{R}{d}}, \tag{42}$$

with a rms error not exceeding 5% [λ_{eff} and $(\lambda_r)_\infty$ are given, respectively, by (2), (27) and (28)]. This formula was obtained in [6] as a result of analytical solution of the system of Eqs. (37)–(40) at $u(r) = u_\infty =$ const. with the use of the concepts on the existence of a gas (liquid) film with the

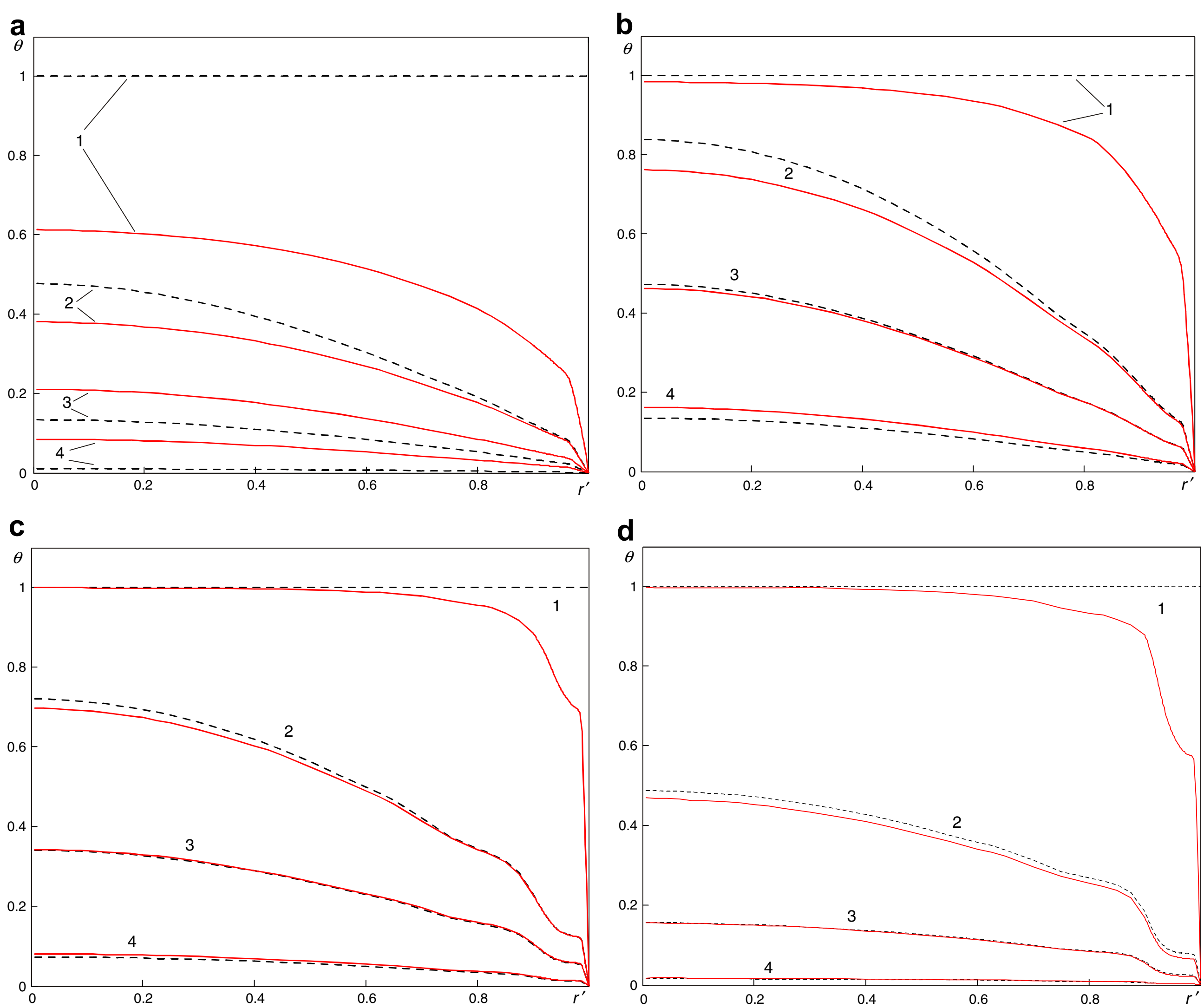

Fig. 3. Temperature distribution in different cross sections of the tube with a granular bed: (a) $Re_\infty = 1$, (b) 10, (c) 100, (d) 500. (1) $x' = 0$, (2) 0.25, (3) 0.5, (4) 1. Solid lines – solution (32)–(36), dashed lines – (37)–(40).

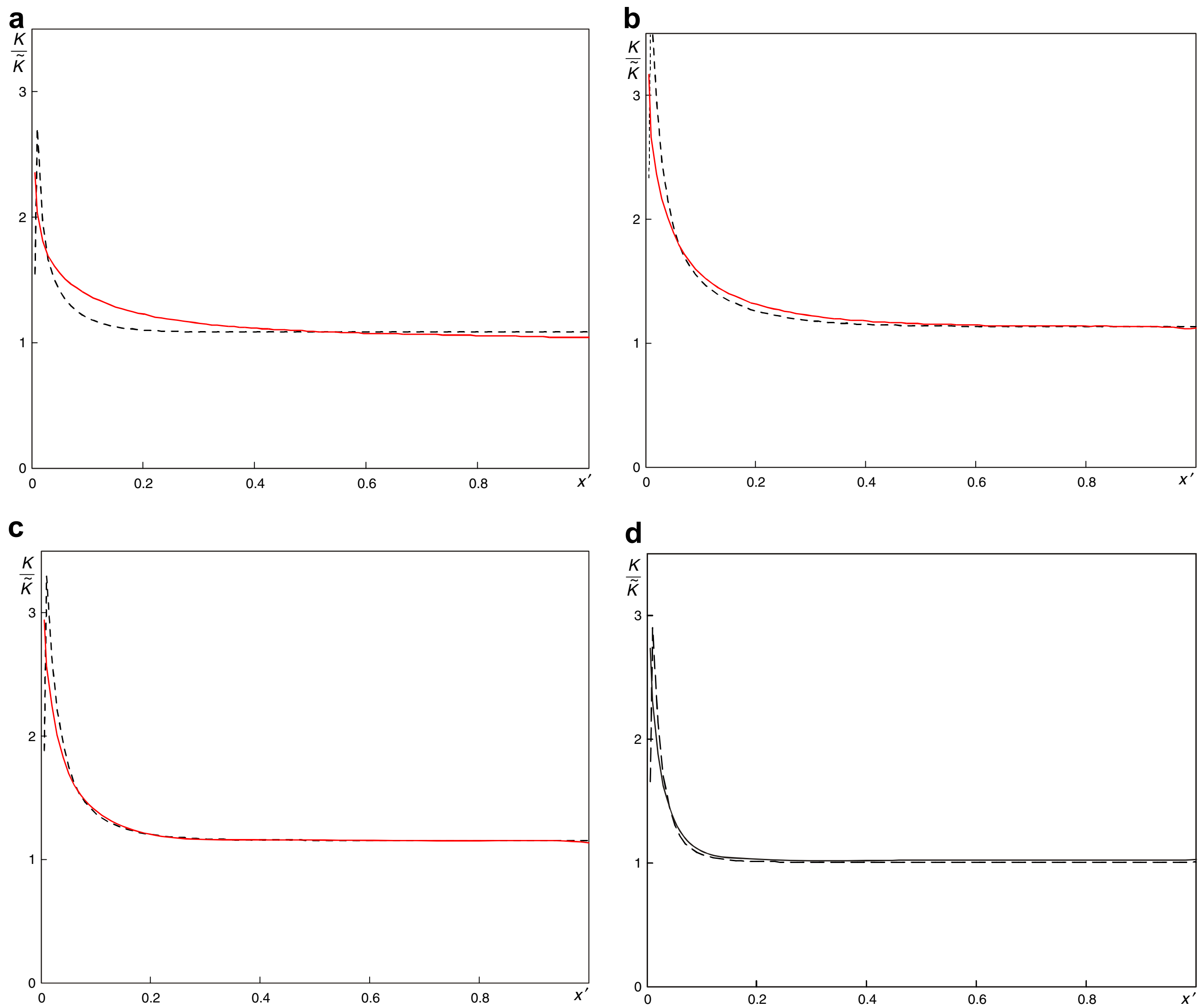

Fig. 4. Variation of the local heat-transfer coefficient along the length of the tube with a granular bed: (a) $Re_\infty = 1$, (b) 10, (c) 100, (d) 500. Solid lines – solution (32)–(36), dashed lines – (37)–(40).

parameters (1) and (2) near the wall. This fact can serve as a basis for using a simple two-layer model (1)–(3) in calculations.

In Fig. 5, the calculation by (42) is compared with the experimental data available in the literature [8,16, Fig. V.24]. As is seen, in all cases, the calculated data agree well with those obtained experimentally. Due to a great dependence of $(\lambda_r)_\infty$ on Re_∞, the contribution of the bed core to the total resistance [the second term in the denominator of (42)] to heat transfer decreases with an increase of velocity.

An important parameter which determined the intensity of the heat transfer process is the value of the initial thermal section x_{st}. For calculation of it, a simple approximation relation

$$x_{st}/L = Re_\infty^{-0.3}(R/d)^{0.4}, \quad 1 \leqslant Re_\infty \leqslant 2000 \tag{43}$$

is obtained. This relation indicates a decrease of x_{st} with an increase of the rate of gas (liquid) filtration. This unusual relation that qualitatively differs from those similar for one-phase media [15] can be explained by a strong dependence of λ_{eff}, λ_r, and λ_a on the rate of filtration which leads to enhancement of heat transfer and decrease of the inlet section with an increase of the rate of filtration.

5. Conclusions

A model of heat transfer in a granular bed ((6), (15), (16), (18), (27)–(31)) has been developed. The model allows for main specific features of the process, i.e., anisotropy of thermal properties of the bed, nonuniform distribution of porosity and filtration rate across the bed. The concepts of the filtration boundary layer and the viscous sublayer have been introduced. Approximation relations have been obtained for calculation of the thicknesses (19)–(20) and thermal sublayer (24)–(26). It was shown that the calculated values of α_{st} are in good agreement with relation (42) obtained in [6] within the framework of the two-layer model of heat transfer (1)–(3). An approximation relation

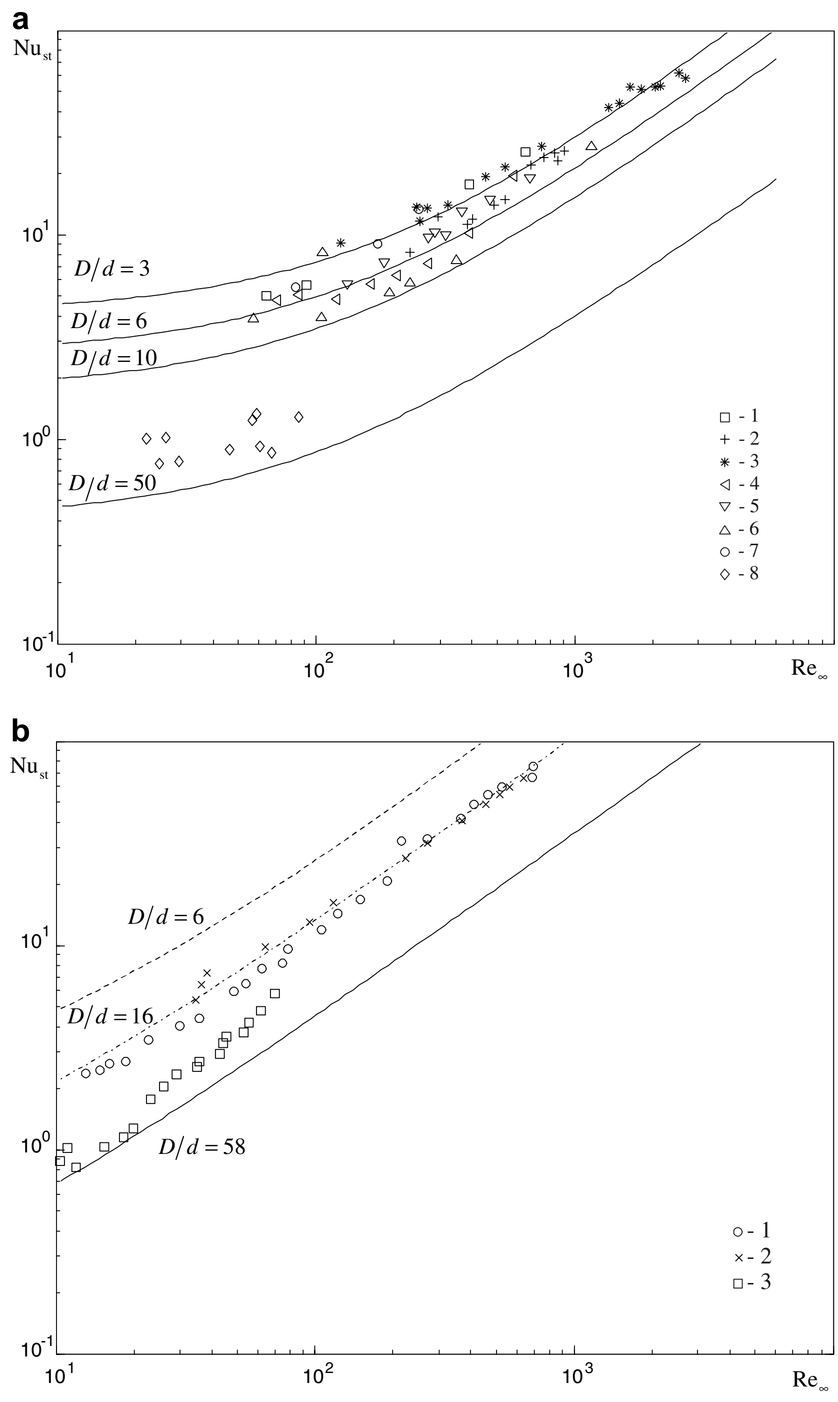

Fig. 5. Heat exchange between the granular bed and the tube wall. Comparison with experimental data: (a) glass spheres–air [16]; (1) $D/d = 5$; (2) 7; (3) 6–9; (4) 7–10; (5) 10; (6) 10–14; (7) 10–14; (8) 40–50; (b) glass spheres–water [8]; (1) $D/d = 6$; (2) 16; (3) 58.

was found for calculation of the inlet thermal section (43). The results of the study can serve as a ground for application of the model of near-wall heat transfer (1)–(3) in the computational practice.

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